Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– IV EXAMINATION – SUMMER 2020

Subject Code: 3140708 **Date:29/10/2020**

Subject Name: Discrete Mathematics

Time: 10:30 AM TO 01:00 PM **Total Marks: 70**

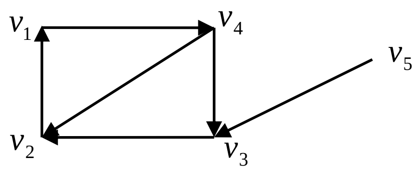
Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	If $A = \{a, b\}$ and $B = \{c, d\}$ and $C = \{e, f\}$ then find *(i)* $(A\times B)U(B\times C)$ $(ii)A\times(BUC)$.	**03**
	(b)	Define even and odd functions. Determine whether the function $f : I \rightarrow R^{+}$ defined by $f(x0 = 2x + 7$ is one-to-one or bijective.	**04**
	(c)	**(i)** Show that the relation $x \equiv y(\bmod m)$ defined on the set of integers I is an equivalence relation.	**03**
		(ii) Draw the Hasse diagram for the partial ordering $\{(A, B)/A \subseteq B\}$ on the power set $P(S)$, where $S = \{a,b,c\}$.	**04**
Q.2	**(a)**	Define equivalence class. Let R be the relation on the set of integers I defined by $(x-y)$ is an even integer, find the disjoint equivalence classes	**03**
	(b)	A committee of *5* persons is to be formed from *6* men and *4* women. In how many ways can this be done when (i) at least *2* women are included (ii) at most *2* women are included ?	**04**
	(c)	Solve the recurrence relation $a_n + 5a_{n-1} + 6a_{n-2} = 3n^2$ using the method of undetermined coefficients.	**07**
		OR	
	(c)	Solve the recurrence relation using the method of generating function $a_n - 5a_{n-1} + 6a_{n-2} = 3^n, n \geq 2; a_0 = 0, a_1 = 2$.	**07**
Q.3	**(a)**	Define simple graph, degree of a vertex and complete graph.	**03**
	(b)	Define tree. Prove that there is one and only one path between every pair of vertices in a tree *T*.	**04**
	(c)	**(i)** A graph *G* has *15* edges, *3* vertices of degree *4* and other vertices of degree *3*. Find the number of vertices in *G*.	**03**
		(ii) Define vertex disjoint and edge disjoint subgraphs by drawing the relevant graphs.	**04**
		OR	
Q.3	**(a)**	Show that $(G,+_5)$ is a cyclic group, where *G={ 0, 1, 2, 3, 4 }*.	**03**
	(b)	Define the following by drawing graphs (i) weak component (ii) unilateral component (iii) strong component.	**04**
	(c)	**(i)** Construct the composite tables for *(i)* addition modulo *4* and *(ii)* multiplication modulo *4* for $Z_4 = \{0,1,2,3\}$. Check whether they have identity and inverse element.	**03**
		(ii) Define ring. Show that the set $M = \left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} / a, b \in R \right\}$ is not a ring under the operations of matrix addition and multiplication.	**04**

Q.4 **(a)** Define algebraic structure, semi group and monoid. Also give related examples. **03**

(b) Use Warshall's algorithm to obtain path matrix from the adjacency matrix of **04**



(c) **(i)** Is the algebraic system *(Q, *)* a group? Where *Q* is the set of rational numbers and * is a binary operation defined by $a * b = a + b - ab,\ \forall a, b \in Q$. **03**

(ii) Let *(Z, +)* be a group, where *Z* is the set of integers and + is an operation of addition. Let *H* be a subgroup of *Z* consisting of elements multiple of *5*. Find the left cosets of *H* in *Z*. **04**

OR

Q.4 **(a)** Prove that there are always an even number of vertices of odd degree in a graph. **03**

(b) Prove that every subgroup H of an abelian group is normal. **04**

(c) **(i)** Find the number of edges in a *r* – regular graph with *n* – vertices. **03**

(ii) A tree *T* has *4* vertices of degree *2, 4* vertices of degree *3, 2* vertices of degree *4*. Find the number of pendant vertices in *T*. **04**

Q.5 **(a)** Show that the operation * defined by $x * y = x^y$ on the set *N* of natural numbers is neither commutative nor associative. **03**

(b) Prove that an algebraic structure *(G, *)* is an abelian group, where *G* is the set of non-zero real numbers and * is a binary operation defined by $a * b = \frac{ab}{2}$. **04**

(c) **(i)** Find out using truth table, whether $(p \wedge r) \rightarrow p$ is a tautology. **03**

(ii) Obtain the dnf of the form $\neg(p \rightarrow (q \wedge r))$. **04**

OR

Q.5 **(a)** If $f : x \rightarrow 2x$, $g : x \rightarrow x^2$ and $h : x \rightarrow (x+1)$, then show that $(fog)oh = fo(goh)$. **03**

(b) Define lattice. Determine whether the POSET $(\{1,2,3,4,5\};|)$ is lattice. **04**

(c) **(i)** If *p:* product is good, *q:* service is good, *r:* company is public limited, write the following in symbolic notations *(i)* either product is good or the service is good or both, *(ii)* either the product is good or service is good but not both, *(iii)* it is not the case that both product is good and company is public limited. **03**

(ii) For the universe of integers, let *p(x), q(x), r(x), s(x)* and *t(x)* be the following open statements*: p(x): x > 0; q(x): x* is even*; r(x): x* is a perfect square; *s(x): x* is divisible by *4*; *t(x): x* is divisible by *5*. Write the following statements in symbolic form: *(i)* At least one integer is even, *(ii)* There exists a positive integer that is even, *(iii)* If *x* is even then *x* is not divisible by *5, (iv)* No even integer is divisible by *5, (v)* There exists an even integer divisible by *5*. **04**